Random Access in Wireless Networks With Overlapping Cells

Gam D. Nguyen, Jeffrey E. Wieselthier, *Fellow, IEEE*, and Anthony Ephremides, *Fellow, IEEE*

***Abstract*—We study cellular-like wireless networks in which the cells may overlap substantially, and a common channel is used for all cells. Thus, transmissions intended for one destination (or base station) can cause interference at neighboring destinations. We assume the use of a "collision-channel" model, in which arbitrary communication and interference regions are associated with each destination. The interaction between such cells is best exemplified if the protocol of access in each cell is pure random access, i.e., Slotted Aloha. We derive a mathematical formula for the maximum achievable throughput for multiple-cell networks that satisfy a "balance" condition, which is related to (but not as stringent as) symmetry. This formula implies that the throughput achieved in a cell is affected only by the degree of overlap with adjacent cells, i.e., a cell's throughput is not affected by transmissions that are outside of its interference region. Moreover, we show that, at the point of maximum throughput, the expected channel traffic is one packet per slot in each cell, an extension of the result obtained many years ago for single-destination networks.**

***Index Terms*—Aloha, cellular, channel traffic, collision channel, multiple destinations, random access.**

I. INTRODUCTION

WE study channel access in wireless networks that consist of multiple overlapping cells. To understand such networks at a fundamental level, we consider a *single* channel that is used in all cells of the networks. This departs from the classical model of assigning distinct frequency channels to adjacent cells. We focus on the uplink channel (i.e., the channel from the users to the destinations). As a consequence of the use of a single channel, the interference from neighboring cells at a destination can be substantial.

Our objective is to study the impact of other-user interference on throughput in multicell networks. We derive the expression for the maximum throughput in heavily loaded networks that use stabilized Slotted Aloha and satisfy the "balance" condition (see Section IV). As expected, the maximum throughput depends on the degree of overlap. We also show that the channel traffic is equal to one packet per slot at each destination at maximum throughput, regardless of the amount of overlap. This is a generalization of the original result derived many years ago for single-destination Slotted Aloha networks [1]–[4].

Manuscript received September 27, 2007; revised June 19, 2009. Current version published May 19, 2010. This work was supported by the U.S. Office of Naval Research.

G. D. Nguyen is with the Information Technology Division, Naval Research Laboratory, Washington, DC 20375 USA (e-mail: gam.nguyen@nrl.navy.mil).

J. E. Wieselthier is with the Wieselthier Research, Silver Spring, MD 20901 USA (e-mail: jeff@wieselthier.com).

A. Ephremides is with the Electrical and Computer Engineering Department and Institute for Systems Research University of Maryland, College Park, MD 20742 USA (e-mail: etony@umd.edu).

Communicated by S. Ulukus, Associate Editor for Communication Networks.

Digital Object Identifier 10.1109/TIT.2010.2046194

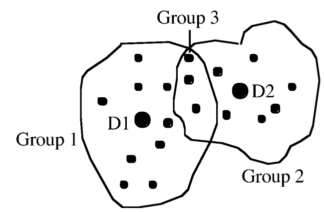

Fig. 1. Multiple access to two destinations by overlapping user populations.

II. INTERFERENCE IN MULTIPLE-DESTINATION RANDOM-ACCESS NETWORKS

We start by considering a simple network with two destinations (D_1 and D_2) and a number of users that are within their *interference* regions (Fig. 1). We assume that each packet is intended for a particular single destination. Successful (i.e., collision-free) reception at the wrong destination does not add to successful throughput. Note that interference region can be greater than communication region and the regions (or cells) can have arbitrary shape.

We say a transmission is "heard" at a destination if its signal strength at that destination is sufficiently large to cause destructive interference (i.e., a collision), although it may not be strong enough to be received successfully (i.e., it may be outside the communication region of the destination). We define group 1 to be the set of users within interference region of only D_1. All of group-1 users are intended for D_1. Similarly, group 2 is the set of users within interference region of only D_2. All of group-2 users are intended for D_2.

Users that are within the intersection of the two interference regions are said to be in group 3. We assume that some group-3 packets are intended for D_1, while other group-3 packets are intended for D_2. Transmissions by group-3 users are heard by both D_1 and D_2, and can cause interference (collisions) with packets for either destination. As in [5], we assume that the destructive interference caused by the combined effect of two or more users outside the interference region is negligible.

The notion of groups is easily extended to networks with N destinations. As in [5], we define groups in terms of the subset of destinations at which transmitted packets in that group are heard. Because there are 2^N-1 nonempty subsets of destinations, there are 2^N-1 possible groups, which are labeled from 1 through 2^N-1. Note that some groups may be empty.

Let the label of a group be denoted by g, which is an integer between 1 and 2^N-1. We then expand g into a binary number

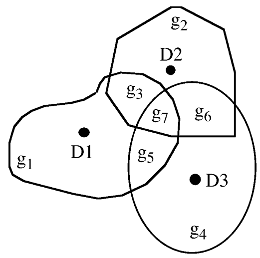

Fig. 2. Three-destination network ($g_i = \text{group } i$).

consisting of N ones and zeroes. A one in the dth position in this binary number indicates that destination d will hear transmissions from group g. A zero in the dth position indicates that destination d will not hear from group g. We now let $E(i)$ be the set of destinations that are within the interference region of group i. That is, $E(i)$ is the set of destinations determined by the 1's in the binary expansion of group i. A complete listing of the sets of destinations corresponding to each group for a three-destination network is (see Fig. 2): $E(1) = \{1\}, E(2) = \{2\}, E(3) = \{1,2\}, E(4) = \{3\}, E(5) = \{1,3\}, E(6) = \{2,3\}$, and $E(7) = \{1,2,3\}$.

III. Slotted Aloha for Multiple-Destination Networks

We now describe the operation of Slotted Aloha in a network with multiple destinations and overlapping user populations. Each newly generated packet is a member of one of the groups defined in the previous section, as determined by the set of destinations at which it can be heard. Moreover, each such packet is intended for one specific destination (which is implicitly assumed to be within its communication range).

The total packet-generation rate (or, equivalently, total arrival rate), summed over all destinations in the network, is designated by λ_t, and is measured in packets per slot. All packets have the same fixed length, which is equal to one time slot [4]. Each newly arriving packet, rather than being transmitted immediately, joins the "backlog." In any arbitrary slot, each backlogged packet is transmitted with a suitable transmission probability (to be specified later in this section). Packets that experience collisions remain in the backlog, and must be retransmitted in a later slot. A packet permanently leaves the network when it is successfully transmitted.

To simplify the discussion, we frequently use the term "users" to refer to *backlogged packets*. That is, a new user is created when a new packet arrives to the network, i.e., we identify this new packet with this new user. This user exists in the network until the packet is successfully transmitted, and then the user disappears.

A. Transmission Probability, Cells, and Channel Traffic

For the Slotted Aloha protocol, the *transmission probability* is defined as the probability with which a backlogged packet is transmitted in a slot. In single-destination stabilized Slotted Aloha, the throughput is maximized when this transmission probability is chosen as $1/m$, where m is the number of backlogged packets. The maximum throughput approaches $1/e$ as m approaches ∞ [2], [3], [6]. Similarly, in the multiple-destination networks studied in this paper, the transmission probability of a packet is chosen based on the total backlog summed over all users that can be heard at the packet's destination (which now also includes packets intended for other destinations), as is discussed below.

Definition 1: The "cell of destination d in slot s," denoted by $C_d(s)$, is defined as the set of backlogged packets that are within the *interference* region of d at the beginning of slot s.

Thus, in this paper, a "cell" refers to a time-varying set of backlogged packets that are within the interference region of a particular destination. For example, the network in Fig. 1 has two cells: $C_1(s)$ is the union of group 1 and group 3, and $C_2(s)$ is the union of group 2 and group 3. Thus, some packets are members of more than one cell.

Let $n(s,i,d)$ be the number of backlogged packets that belong to group i and are intended for destination d in time slot s. We assume that these $n(s,i,d)$ packets are transmitted with the same transmission probability, denoted by $p(s,i,d)$. Further, we require that

$$p(s,i,d) = p(s,j,d) \tag{1}$$

i.e., all packets that are intended for the same destination are transmitted with the same probability, even if they belong to different groups. We denote the common transmission probability in (1) by $p_d(s)$.

Let $G(s,i,d)$ be the *channel traffic* at destination d (i.e., the average number of transmissions per slot) when the backlog in slot s is $n(s,i,d)$ packets. We have

$$G(s,i,d) = n(s,i,d)p_d(s). \tag{2}$$

Let $G_{\text{heard}}(s,d)$ be the channel traffic caused by the transmitted packets in cell $C_d(s)$, i.e.,

$$G_{\text{heard}}(s,d) = \sum_{i=1}^{2^N-1} \sum_{e=1}^{N} G(s,i,e).$$

Let $f(i,d)$ be the *fraction* of the packet arrivals that belong to group i and are intended for destination d, i.e.,

$$f(i,d) = \lambda(i,d)/\lambda_t \tag{3}$$

where λ_t is the total arrival rate for the network, given by

$$\lambda_t = \sum_{i=1}^{2^N-1} \sum_{d=1}^{N} \lambda(i,d) \tag{4}$$

and $\lambda(i,d)$ is the arrival rate of packets belonging in group i and intended for destination d.

Let $f_{\text{heard}}(d)$ be the fraction of arriving packets that are within the interference region of destination d, and $f_{\text{intended}}(d)$ be the fraction of arriving packets that are intended for d. We have

$$f_{\text{intended}}(d) = \sum_{i=1}^{2^N-1} f(i,d)$$

and

$$f_{\text{heard}}(d) = \sum_{i=1}^{2^N-1} \sum_{j=1}^{N} f(i,j). \tag{5}$$

IV. Maximum Throughput of Balanced Networks

For the classic case of a single-cell network (in which the only cell is denoted by $C_1(s)$) with a large number of backlogged packets, stabilized Slotted Aloha's maximum throughput of $1/e$ packets/slot is achieved when the transmission probability is $p_1(s) = 1/|C_1(s)|$ [3], [6]. Thus, the transmission probability is inversely proportional to the number of packets in cell $C_1(s)$. In practice, this number may not be precisely known and may be estimated by, for example, the method in [6]. Motivated by this fact, for the case of multiple destinations, we implement stabilized Slotted Aloha for cell $C_d(s)$ by letting the transmission probability for each packet intended for destination d (which is a subset of the packets in $C_d(s)$) be inversely proportional to the total number of backlogged packets in $C_d(s)$.

Because cells overlap, a backlogged packet may be a member of several cells. Its transmission probability is based on $|C_d(s)|$, which includes packets in $C_d(s)$ that are intended for other destinations, in addition to those intended for d. That is, in slot s, we require that each packet intended for destination d transmit with probability $p_d(s) = a_d/|C_d(s)|$ for some constant a_d, which we determine later.

Let us now derive the maximum throughput at an arbitrary destination d. Thus, consider an arbitrary time slot s, and let us focus our attention on the cell $C_d(s)$, because packets that are not in $C_d(s)$ cannot interfere at destination d. We can write

$$|C_d(s)| = n_{d,d}(s) + \sum_{e \neq d} n_{d,e}(s)$$

where $n_{d,e}(s)$ is the number of packets in cell $C_d(s)$ that are intended for destination e. Let $N_{d,e}(s)$ be the random variable representing the number of transmitted packets in $C_d(s)$ that are intended for destination e.

Recall that all the packets in $C_d(s)$ that are intended for d are transmitted with the same transmission probability $p_d(s) = a_d/|C_d(s)|$. A transmission is successful at d if (a) there is exactly one transmission of a packet in $C_d(s)$ that is intended for d, and (b) there are no transmissions of packets in $C_d(s)$ that are not intended for d. Thus, the throughput (i.e., the probability of successful transmission) at d is given by

$$S_d(s) = \Pr\{N_{d,d}(s) = 1\} \prod_{e \neq d} \Pr\{N_{d,e}(s) = 0\} \tag{6}$$

where

$$\Pr\{N_{d,d}(s) = 1\} = n_{d,d}(s) \frac{a_d}{|C_d(s)|} \left(1 - \frac{a_d}{|C_d(s)|}\right)^{n_{d,d}(s)-1}$$

and

$$\Pr\{N_{d,e}(s) = 0\} = \left(1 - \frac{a_e}{|C_e(s)|}\right)^{n_{d,e}(s)}$$

for $e \neq d$. From (6), we have

$$S_d(s) = n_{d,d}(s) \frac{a_d}{|C_d(s)|} \left(1 - \frac{a_d}{|C_d(s)|}\right)^{n_{d,d}(s)-1} \times \prod \left(1 - \frac{a_e}{|C_e(s)|}\right)^{n_{d,e}(s)}. \tag{7}$$

In the following, we consider a special class of networks for which we are able to derive the exact formula for the maximum throughput. First, note that the total number of backlogged packets in the network at the beginning of slot s is

$$n_t(s) = \sum_{i=1}^{2^N-1} \sum_{d=1}^{N} n(s,i,d).$$

Definition 2: We say that a network with N destinations is *balanced* in time slot s if the following conditions are satisfied for each destination d and each group i:

1) $f_{\text{heard}}(1) = f_{\text{heard}}(2) = \cdots = f_{\text{heard}}(N)$,
2) $p_d(s) = a/|C_d(s)|$, where a is a constant, and
3) $\lim_{n_t(s)\to\infty} \frac{n(s,i,d)}{n_t(s)} = f(i,d)$.

Remark 1:

1) Condition 1 for balanced networks means all cells experience the same level of average offered traffic load. Networks that satisfy this condition are easy to construct (see Section V). Condition 2 states that the same constant a is used in $p_d(s)$ for all destinations d. This condition restricts the class of networks for which we can provide a definite result (see Theorem 1). However, it is in harmony with the approaches used for single destination networks, and can be considered as an interesting special case that employs uniform transmission control.
2) Recall from (3) that $f(i,d) = \lambda(i,d)/\lambda t$, where λ_t is the total arrival rate for the network, given by (4). Thus, Condition 3 of Definition 1 is equivalent to
$$\lim_{n_t(s)\to\infty} \frac{n(s,i,d)}{n_t(s)} = \frac{\lambda(i,d)}{\lambda_t}.$$
3) Condition 3 requires $n_t(s) \to \infty$. (i) One (obvious but impractical) method to ensure that $n_t(s) \to \infty$ is by letting the average number of packets arriving into the network per time slot to approach infinity, i.e., $\lambda_t \to \infty$. With this method, we have $n_t(s) \to \infty$ for every slot s. Further, from a strong law of large numbers, it can be shown that $n(s,i,d)/n_t(s) \to f(i,d)$ with probability 1, i.e., Condition 3 of Definition 1 holds. (ii) Another (experimental) method is to choose an arrival rate λ_t slightly larger than the total throughput (i.e., the departure rate), so that the number of backlogged packets steadily increases as time progresses. With this method, we have $n_t(s) \to \infty$ as $s \to \infty$. Our simulations for a variety of networks indicate that Condition 3 holds as long as both Conditions 1 and 2 hold.
4) For balanced networks, $f(i,d)$ depends on $n_t(s)$ through Condition 3. Thus, we have
$$\lim_{n_t(s)\to\infty} \frac{n(s,i,d)}{n(s,j,e)} = \lim_{n_t(s)\to\infty} \frac{n(s,i,d)/n_t(s)}{n(s,j,e)/n_t(s)} = \frac{f(i,d)}{f(j,e)}$$
for all groups i, j, and all destinations d, e, such that $f(j,e) > 0$. Similarly, it can be shown from Conditions 2 and 3 that $p_e(s)/p_d(s) = |C_d(s)|/|C_e(s)| \to 1$ as $n_t(s) \to \infty$.

5) For a single-destination network (which clearly satisfies our definition of balance), it is well known that the maximum throughput is achieved when the channel traffic (i.e., cell traffic) is one packet per slot [2], [3]. The following theorem shows that the same conclusion also holds for balanced multiple-destination networks. Note that the theorem is valid for both symmetric and asymmetric networks (see Section V).

Theorem 1: For a *balanced* network that contains a very large number of backlogged packets in time slot s, i.e., $n_t(s) \to \infty$, the maximum throughput at destination d is given by

$$S_d(s) = \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} e^{-1}$$

which is achieved if and only if $p_d(s) = 1/|C_d(s)|$ and the channel traffic is one packet per slot, i.e., $G_{\text{heard}}(s,d) = 1$.

Proof: Using Condition 1 of Definition 1 in the general throughput expression (7), we have

$$\begin{aligned} S_d(s) &= n_{d,d}(s)\frac{a}{|C_d(s)|}\left(1-\frac{a}{|C_d(s)|}\right)^{n_{d,d}(s)-1} \\ &\quad \times \prod_{e\neq d}\left(1-\frac{a}{|C_e(s)|}\right)^{n_{d,e}(s)} \\ &= n_{d,d}(s)\frac{a}{|C_d(s)|}\left(1-\frac{a}{|C_d(s)|}\right)^{n_{d,d}(s)-1} \\ &\quad \times \prod_{e\neq d}\left(\frac{a}{|C_d(s)|}\left[\frac{|C_d(s)|}{a}-\frac{|C_d(s)|}{|C_e(s)|}\right]\right)^{n_{d,e}(s)}. \end{aligned}$$

By letting $n_t(s) \to \infty$, we have $|C_d(s)|/|C_e(s)| \to 1$ (by Remark 1.4), and, hence

$$\begin{aligned} S_d(s) &\to n_{d,d}(s)\frac{a}{|C_d(s)|}\left(1-\frac{a}{|C_d(s)|}\right)^{n_{d,d}(s)-1} \\ &\quad \times \prod_{e\neq d}\left(\frac{a}{|C_d(s)|}\left[\frac{|C_d(s)|}{a}-1\right]\right)^{n_{d,e}(s)} \\ &= n_{d,d}(s)\frac{a}{|C_d(s)}\left(1-\frac{a}{|C_d(s)|}\right)^{n_{d,d}(s)-1} \\ &\quad \times \prod_{e\neq d}\left(1-\frac{a}{|C_d(s)|}\right)^{n_{d,e}(s)} \\ &= n_{d,d}(s)\frac{a}{|C_d(s)|}\left(1-\frac{a}{|C_d(s)|}\right)^{|C_d(s)|-1} \\ &= \frac{\sum_{i=1}^{2^N-1} n(s,i,d)}{\sum_{i=1,d\in E(i)}^{2^N-1}\sum_{j=1}^{N} n(s,i,j)} a\left(1-\frac{a}{|C_d(s)|}\right)^{|C_d(s)|-1} \\ &\to \frac{\sum_{i=1}^{2^N-1} f(i,d)}{\sum_{i=1,d\in E(i)}^{2^N-1}\sum_{j=1}^{N} f(i,j)} ae^{-a} = \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} ae^{-a}. \end{aligned}$$

To summarize, for the balanced network with an infinite number of backlogged packets, we have

$$S_d(s) = \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} ae^{-a}.$$

Next, we have

$$\begin{aligned} G_{\text{heard}}(s,d) &= p_d(s)n_{d,d}(s) + \sum_{e\neq d} p_i(s)n_{d,e}(s) \\ &= p_d(s)\left(n_{d,d}(s) + \sum_{e\neq d}\frac{p_i(s)}{p_d(s)}n_{d,e}(s)\right). \end{aligned}$$

For the balanced network with $n_t(s) \to \infty$, we have $p_e(s)/p_d(s) \to 1$ by Remark 1.4. Thus

$$G_{\text{heard}}(s,d) \to p_d(s)|C_d(s)| = a.$$

In summary, we have $G_{\text{heard}}(s,d) = a$ for the balanced network with an infinite number of packets. Similarly, it can be shown that $G_{\text{heard}}(s,e) = a$ for any slot s and any destination e.

It is well known that $xe^{-x} \leq e^{-1}$, and the equality is achieved if and only if $x = 1$. Using this fact in the above throughput expression, we have

$$S_d(s) \leq \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} e^{-1}$$

in which the equality is achieved (i.e., when $S_d(s)$ is maximized) if and only if $a = 1$. Thus, the maximum throughput at destination d is

$$S_d(s) = \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} e^{-1}$$

which is achieved if and only if $G_{\text{heard}}(s,d) = 1$ (i.e., $p_d(s) = 1/|C_d(s)|$). ∎

Remark 2:

1) From Theorem 1, the maximum throughput and the channel traffic do not depend on the time slot variable s, as long as $n_t(s) \to \infty$. Thus, for balanced networks with infinite backlog, we can omit s from the expressions for maximum throughput and the channel traffic, i.e., we simply write S_d and $G_{\text{heard}}(d)$ instead of $S_d(s)$ and $G_{\text{heard}}(s,d)$.
2) In Theorem 1, the throughput at destination d depends only on $f_{\text{intended}}(d)$ and $f_{\text{heard}}(d)$. Thus, for multiple-destination balanced networks, the throughput computed at a cell does not depend on the transmissions at other cells that do not overlap with it. That is, a cell's throughput is not affected by users that are outside of its interference region.
3) For balanced networks, the following facts follow directly from Theorem 1. The total network throughput is given by

$$S_{\text{total}} = \sum_{d=1}^{N} S_d = \frac{e^{-1}}{f_{\text{heard}}(1)}.$$

When the cells are disjoint, we have $S_d = e^{-1}$. When the cells overlap fully, we have $S_d = e^{-1}/N$.

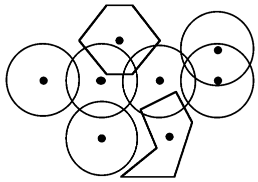

Fig. 3. General network configuration.

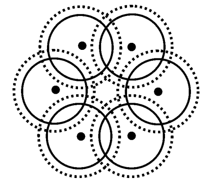

Fig. 4. Symmetric six-destination network.

V. Examples of Balanced Multiple-Destination Networks

We now present examples of networks for which $f_{\text{heard}}(d) = f_{\text{heard}}(1)$ for all destinations d. Here, for each destination d and each time slot s, we assume that $p_d(s) = 1/|C_d(s)|$ and $n(s,i,d)/n_t(s) \to f(i,d)$ as $n_t(s) \to \infty$ (see Remark 1.3). That is, these networks are balanced (by Definition 1), and, hence, the maximum throughput at each destination can be analytically computed (using Theorem 1). In the following, we assume that the arrivals are uniformly distributed spatially throughout the network (i.e., throughout the union of all communication regions). Therefore, the arrival rate corresponding to any such region is proportional to its area.

Fig. 3 shows an example of a general network with N destinations (or base stations), shown by black dots. The interference region associated with each destination d is also shown, $1 \leq d \leq N$. In this example, we assume that interference regions are equal to the communication regions for all destinations. It is clear that a sufficient condition for $f_{\text{heard}}(d) = f_{\text{heard}}(1)$, for all d, is that the interference regions of all destinations are equal in size (although they can be of arbitrary shape). Note that the throughput of any particular cell depends on the degree of overlap from neighboring cells, as implied by Theorem 1, where $f_{\text{intended}}(d)$ and $f_{\text{heard}}(d)$ are evaluated based on geometry.

To illustrate geometrical considerations that lead to balanced networks, we now consider some examples in which the interference regions are larger than the communication regions. For example, consider a finite cellular network in which the destinations are located on a square grid, communication and interference regions are circular, the interference regions of adjacent cells overlap, and the arrivals are limited to a region that consists of the union of the communication regions. In such cases, the interference regions of the "edge" cells and "corner" cells will contain regions with no arrivals. Consequently, the balance condition would be violated, even when the communication range is the same for each cell and the interference range is the same for each cell.

However, there is a special class of networks, which we call *symmetric* networks, that continue to satisfy the balance condition even when the interference regions are larger than the communication regions. Recall from Section II that $E(i)$ denotes the set of destinations that are within the interference region of group i. Let w_i be the *weight* of group i, i.e., the number of ones that are contained in the binary expansion of i.

Definition 3: A multiple-destination network is said to be *symmetric* if $f(i,d) = f(j,e)$ for all groups i, j, and all destinations d, e, such that $w_i = w_j$, $d \in E(i)$, and $e \in E(j)$.

For symmetric networks, it is easily observed (on the basis of geometry and the fact that arrivals are uniformly spatially distributed) that $f_{\text{heard}}(d) = f_{\text{heard}}(1)$ and $f_{\text{intended}}(d) = f_{\text{intended}}(1)$ for all destinations d, i.e., Condition 1 of Definition 1 holds. Again, we also assume that Conditions 2 and 3 hold. Thus, symmetric networks are balanced, and additionally the throughput is the *same* for all destinations.

Consider the two-destination network shown in Fig. 1. Assume that this network is symmetric. Definition 3 implies that $f(1,1) = f(2,2)$ and $f(3,1) = f(3,2)$. Note that $f(2,1) = f(1,2) = 0$ and $f(1,1) + f(3,1) = 0.5$. From Theorem 1, we have

$$S_d = \frac{f_{\text{intended}}(d)}{f_{\text{heard}}(d)} e^{-1} = \frac{f(1,1) + f(3,1)}{f(1,1) + 2f(3,1)} e^{-1} = \frac{0.5}{0.5 + f(3,1)} e^{-1} = \frac{1}{1 + 2f(3,1)} e^{-1}$$

i.e., the throughput per destination depends on only $f(3,1)$. This agrees with the results in [5].

Symmetric networks of arbitrary number of destinations can be configured, e.g., by placing the destinations on the vertices of regular polygons, as shown in Fig. 4 for a six-destination network. The users are located inside the solid circles whose radii represent the common communication range associated with each destination. The dotted circles represent the interference regions, which are also assumed to be the same size for all destinations.

VI. Conclusion

This paper has addressed the random-access problem in networks with multiple destinations with overlapping user populations, using a collision-channel model. For stabilized Slotted Aloha, under the "balance" requirement defined in this paper, we are able to analytically compute the maximum throughput at each destination in the network. We have shown that the channel traffic is one packet per slot at each destination when operating at the point of maximum throughput. The throughput of "symmetric" networks, in which the traffic demands are distributed uniformly in space, is easily computed on the basis of geometrical considerations, even when the interference range exceeds the communication range. Our model is motivated by a desire to "push" the limits of sharing a common channel by allowing considerable overlaps among user populations that are intended for different destinations. Our formulation provides the means of capturing the effects of arbitrary levels of interference without the complexities of SINR-based or carrier-sensing models. The intention is to assess the performance loss that occurs as such cells get squeezed closer together for improved spectrum re-use.

REFERENCES

[1] N. Abramson, "The ALOHA system—Another alternative for computer communications," in *Proc. Fall Joint Computer Conf., AFIPS Press*, 1970, vol. 37, pp. 281–285.

[2] V. Ahuja, *Design and Analysis of Computer Communication Networks*. New York: McGraw-Hill, 1982.

[3] D. Bertsekas and R. Gallager, *Data Networks*, 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 1992.

[4] L. G. Roberts, "ALOHA packet system with and without slots and capture," *Comput. Commun. Rev.*, vol. 5–2, pp. 28–42, Apr. 1975.

[5] M. Sidi and I. Cidon, "A multi-station packet-radio network," *Perf. Eval.*, vol. 8, pp. 65–72, 1988.

[6] R. L. Rivest, "Network control by Bayesian broadcast," MIT/LCS/TM-285, MIT Laboratory for Computer Science, 1985.

Gam D. Nguyen received the Ph.D. degree in electrical engineering from the University of Maryland, College Park, in 1990.

He is currently at the Naval Research Laboratory, Washington, DC. His research interests include communication systems and networks.

Dr. Nguyen won the IEEE Fred W. Ellersick Award for the best unclassified paper at MILCOM 2000.

Jeffrey E. Wieselthier (S'67–M'69–SM'88–F'07) was born in Brooklyn, NY, in 1949. He received the B.S. degree from the Massachusetts Institute of Technology, Cambridge, in 1969, the M.S. degree from the Johns Hopkins University, Baltimore, MD, in 1971, and the Ph.D. degree from the University of Maryland, College Park, in 1979, all in electrical engineering.

He was employed at the Naval Surface Warfare Center, White Oak, Silver Spring, MD, from 1969 to 1979. From 1979 to 2007, he was with the Information Technology Division of the Naval Research Laboratory, Washington, DC, where he was a Senior Researcher and Head of the Wireless Network Theory Section of the Networks and Communication Systems Branch. Additionally, he was a program manager in communications and networking for the Office of Naval Research. He is currently a self-employed consultant with Wieselthier Research.

Dr. Wieselthier was Lead Guest Editor of the 2005 two-part special issue on Wireless Ad Hoc Networks that appeared in the *IEEE Journal on Selected Areas in Communications* and he was on the Editorial Board of Elsevier's journal *Ad Hoc Networks*. He was Technical Program Co-Chair of the Third IEEE Symposium on Computers and Communications in Athens, Greece, in 1998 and Treasurer of the 1991 IEEE International Symposium on Information Theory in Budapest, Hungary. He won the IEEE Fred W. Ellersick Award for the best unclassified paper at MILCOM 2000. He has studied a variety of communication networking problems, including multiple access and routing in spread-spectrum networks and the use of neural networks and other approaches for network performance evaluation, optimization and control. His current interests include wireless communication networks, with an emphasis on issues relating to energy-aware, cross-layer, and cooperative operation of ad hoc and sensor networks. He is a member of Eta Kappa Nu and Sigma Xi.

Anthony Ephremides (S'68–M'71–SM'77–F'84) received the Ph.D. degree in electrical engineering from Princeton University, Princeton, NJ, in 1971.

He holds the Cynthia Kim Professorship of Information Technology at the Electrical and Computer Engineering Department of the University of Maryland in College Park, where he holds a joint appointment at the Institute for Systems Research, of which he was among the founding members in 1986. He has held various visiting positions at other Institutions (including MIT, UC Berkeley, ETH Zurich, INRIA, etc.) and cofounded and codirected a NASA-funded Center on Satellite and Hybrid Communication Networks in 1991. He has been the President of Pontos, Inc., since 1980 and served as President of the IEEE Information Theory Society in 1987 and as a member of the IEEE Board of Directors in 1989 and 1990. He has been the General Chair and/or the Technical Program Chair of several technical conferences (including the IEEE Information Theory Symposium in1991 and 2000, the IEEE Conference on Decision and Control in 1986, the ACM Mobihoc in 2003, and the IEEE Infocom in 1999). He has served on the Editorial Board of numerous journals and was the Founding Director of the Fairchild Scholars and Doctoral Fellows Program, a University-Industry Partnership from 1981 to 1985. He is the author of several hundreds of papers, conference presentations, and patents, and his research interests lie in the areas of communication systems and networks and all related disciplines, such as information theory, control and optimization, satellite systems, queueing models, signal processing, etc. He is especially interested in wireless networks and energy efficient systems.

Dr. Ephremides received the IEEE Donald E. Fink Prize Paper Award in 1991 and the first ACM Achievement Award for Contributions to Wireless Networking in 1996, as well as the 2000 Fred W. Ellersick MILCOM Best Paper Award, the IEEE Third Millennium Medal, the 2000 Outstanding Systems Engineering Faculty Award from the Institute for Systems Research, and the Kirwan Faculty Research and Scholarship Prize from the University of Maryland in 2001, and a few other official recognitions of his work. He also received the 2006 Aaron Wyner Award for Exceptional Service and Leadership to the IEEE Information Theory Society.